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DECISION MAKING UNDER UNCERTAINTY (DMUU) — LIMITATION OF CLASSICAL DECISION THEORY

The article describes the problem of applying the classical theory of decision making in the socio-cultural sphere under conditions of uncertainty DMUU). The usage of four decision making criteria under uncertainty of Laplace, Hurwitz, Wald and Savage are compared. It is shown that different criteria are optimal for different situations. Typical limitations of mathematical, methodological and pragmatic nature of the classical theory of decision making are identified and described. The idea that the use of the results will increase the degree of certainty in decision making is substantiated.

Key words: decision making, the decision-maker, criteria, uncertainty.

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ПРИЙНЯТТЯ РІШЕНЬ В УМОВАХ НЕВИЗНАЧЕНОСТІ (DMUU) — ОБМЕЖЕННЯ КЛАСИЧНОЇ ТЕОРІЇ ПРИЙНЯТТЯ РІШЕНЬ

Розглядається проблема застосування класичної теорії прийняття рішень у соціально-культурній сфері в умовах невизначеності (DMUU). Порівнюється використання чотирьох критеріїв прийняття рішень в умовах невизначеності Лапласа, Гурвіца, Вальда і Севіджа. Показано, що різні критерії оптимальні для різних ситуацій. Типові математичні, методологічні та прагматичні обмеження класичної теорії прийняття рішень визначені й описані. Обґрунтовується ідея про те, що використання одержаних результатів дозволить підвищити ступінь визначеності під час прийняття рішень.

Ключові слова: прийняття рішень, критерії, невизначеність.

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ПРИНЯТИЕ РЕШЕНИЙ В УСЛОВИЯХ НЕОПРЕДЕЛЕННОСТИ (DMUU) — ОГРАНИЧЕНИЕ КЛАССИЧЕСКОЙ ТЕОРИИ ПРИНЯТИЯ РЕШЕНИЙ

Рассматривается проблема применения классической теории принятия решений в социально-культурной сфере в условиях неопределенности (DMUU). Дается сравнение использования четырех критериев принятия решений в условиях неопределенности: Лапласа, Гурвица, Вальда и Сэвиджа. Показано, что различные критерии оптимальны для различных ситуаций. Определены и описаны характерные ограничения математического, методологического и прагматического

характера классической теории принятия решений. Обосновывается мысль о том, что использование полученных результатов позволит увеличить степень определенности в принятии решений.
Ключевые слова: принятие решений, критерии, неопределенность.

One basic problem of most people is making rational decisions in the face of uncertainty or incomplete knowledge as to the consequences of one's actions. Meeting uncertainty is a pervasive problem which appears in many areas of human endeavors. How to reduce uncertainty, how much to reduce it before acting, and what actions are reasonable in the face of uncertainty are the basic issues to be examined in this article.

Decision making in the class of problem situations that we would like to consider is in activity in which experiments are difficult to control, most interesting aspects cannot be measured precisely, general laws are completely lacking, each decision problem is viewed as unique, and the intuitive or «judgmental» application of subjective experience is the rule. To understand these problems, it is helpful to have as an overview, a conceptual structure, or model which will help to organize our ideas about decision making. This model considers a decision making as a process involving such steps as:

- analysis of the decision situation; that is, scanning the environment with the aim of recognizing and conceptualizing the decision problem, using both past experience and presently available information,
- deriving and implementing the decision,
- learning from the results of the decision how it should be modified and adding this to the knowledge to the reservoir of experience on which future decisions may draw.

The process is thus one of deciding, acting, and learning from the resulting experience how to act more effectively in the future. It pictures decision making as a dynamic process and suggests that one may regard decision making as the mechanism by which learning takes place.

We would like to formalize problem solving where decisions are made through determining whether or not an event fits in a certain «pattern" — by adding up evidence obtained from many small experiments or observations.

This clear and simple concept is important because most, and perhaps all, more complicated decision systems share a little of this character.

We shall assume that the decision problem is relatively well defined and somewhat repetitive in nature. In the real world many complex decision problems correspond to this paradigm. For example, control of economic processes, medical diagnostics and therapy, investment decision making, etc., all belong to this category.

Decision theory, also, can be used by management in the socio-cultural sphere for a variety of different decisions, including concert activities, organization of leisure, entertainment and so on, and also the activity that provides funding, logistics, training and retraining of personnel, information of support, location planning, production and service design, equipment selection.

In the present study we would like to consider an approach for handling this class of decision problem. But in doing so we shall make maximum use of existing concepts and past experience. Thus, we shall briefly outline the present state of the art in theoretical foundations underlying choices under uncertainty.

A person is faced with a decision problem if he is given:

- an objective or goal,
- a set of possible alternatives for achieving the objective,
- a test for verifying whether a given alternative is in fact a solution to his problem.

The test is usually accomplished by means of a performance criterion or an objective function.

The decision situation or state of nature is characterized by an information structure x. We define the state of a system (nature) at any given time as the information required to determine the behavior of the system from that time on. Sometimes we call x the space vector and the components of x are state variables, while the space X spanned by the vector x is called the space state. In most general terms the vector x is the information needed for decision making. For our purposes x is usually either a set of relevant problem features, or a set observables characterizing decision situation, etc. All uncontrollable decision variables are also contained in x.

Let the symbol A denote the set of possible alternatives: $A = \{a_i\}$, $i = 1,\ldots,m$. Broadly speaking, the decisions ai include the configuration of all controllable variables in the system.

We define formally the decision problem $D (A, X)$ as the task of selecting the best alternative $a_i \in A$ corresponding to a given decision situation characterized by the information $x_j \in X$. Decision making, then, is the activity of solving the decision problems or the decision rule.

The decisions are supposed to be connected in some way to a set of possible outcomes. The choice among outcomes reflects a value judgment. Values must be associated with various outcomes which may result from the decision. Outcomes must be compared as to value, and a decision implies such a comparison; all outcomes must be compared to a common value scale.

The numerical value is assigned to outcomes by means of an index of performance or the objective function of the system. Thus to such recognized action-state pair (*ai, xj*) a value uij is assigned by the objective function $U(A, X)$. The values u_{ij} are also called «payoff» or «utility» of the decision a_i when state of nature is x_j. We are now in a position to conceptualize the decision process. The abstract representation of the decision model is given be the equation

$$u_{ij} = U(a_i, x_j). \quad (1)$$

The procedure for deriving the decision from the decision model is the decision rule introduced below.

There are three different elements that should be considered in decision making: list of alternatives, known payoff (utility) for each alternative, and a set of possible future conditions for each alternative. There are three basic environments in which decisions need to be made:

- tasks under conditions of certainty;
- tasks under conditions of a probabilistic certainty (risk);
- tasks under conditions of uncertainty.

In the first case there is decision making in deterministic situations — it is assumed that all relevant information about the decision situation is known and there is a known deterministic connection between every decision and the corresponding outcome. The states of nature are specified and the outcomes of actions are assumed to be known in advance. Such decision problems are easily programmable, e.g., by decision table methods. But relatively few real-life decision problems fall into this category.

The second branch of decision theory deals with decision making under risk. Many problems of the real world can be formulated as risk problems. In this case the true states of nature or outcomes of one's actions are not known. However, the decision maker has some partial knowledge which can be expressed in terms of probabilities applicable to all states of nature or possible outcomes of actions. Most of classical decision theory deals with decision making under risk.

In many, perhaps most decision problems in the real world, the probability laws characterizing the decision situation are not known beforehand. In this case the decision maker is faced with uncertainty about the possible outcomes of his actions. The information needs for the design of optimal decision systems is not known a priori, or is only incompletely known and, in fact, for various reasons there may be no opportunity to acquire this information in advance. For example, the theory may not well understood, the effects of certain factors may not be known, the form of the functional dependence of performance upon some decision parameters cannot be estimated in advance, and so forth.

However, analysis have devised some decision rules to impart some objectively to the subjective decisions, provided decision-makers are able to identify the possible 'states of nature'. Therefore, decisions taken under uncertainty are necessarily subjective and can estimate the outcome of each strategy. Some such important decision rules are discussed further. decision rules are discussed below

The four decision criteria that have been most seriously considered when facing uncertainty are the Laplace, Hurwicz α, Wald's minimax (or maximin), and Savage's minimal regret. All of these decision criteria assume the knowledge of a decision model $U(a, x)$, which assigns a unique utility value u_{ij} to the outcome of taking action ai when the state of nature is x_j. It is usually also assumed that nature is indifferent toward the decision maker.

Laplace criterion (criterion of the mean) assigns an equal probability to all states of nature. Then the choice would fall to that course of action whose simple arithmetic average (or sum) of the utility values of outcomes is greatest. The basis of this criterion lies the «principle of insufficient reason».

Laplace criterion is also called as law of equal probabilities criterion or criterion of rationality, since probability of states of nature are not known it is assumed that all states of nature will occur in equal probability. i.e. assign an equal probability $q_1 = q_2 = \ldots = q_n = 1/n$ and the Laplace criterion takes the form

$$L = \max_{i} 1/n \cdot \sum_{j} u_{ij} . \qquad (2)$$

Hurwicz α criterion (optimism- pessimism coefficient). The subjectively determined index α assigns predetermined relative weights to the best and the worst of the possible results of each decision. That choice is taken which then has the highest weighted average of the best and worst of the possible results that can stem from a given decision

$$H = \max_{i} [\alpha \min_{j} u_{ij} + (1 - \alpha) \max_{j} u_{ij}]. \qquad (3)$$

The Hurwicz α criterion is a criterion for decision making under complete uncertainty that represents a compromise between the Maximin and Maximax criteria. The α is a number between 0 and 1. In the special case where it is one, the criterion reduces to Maximin and in the special case where it is zero the criterion reduces to Maximax. The decision maker can set α to a number between zero and one according to his or her level of optimism.

By «Decision Making Under Complete Uncertainty» it is meant that a decision table is available. This means that it is known which alternatives are available, which states of nature are possible, and what utility each alternative would derive in each possible state of nature. The «complete » means that the probabilities of each state of nature occurring are unknown.

Wald criterion (maximax or maximin) is derived from Hurwicz criterion by setting α = 0 or 1. In the former case we act as extreme optimists; in the latter case extreme pessimists. The maximin criterion (α = 1) is a conservative approach for a decision which looks at the worst possible outcome for each alternative and selects that course of action which assures the best results for the worst conditions:

$$W = \max_i \min_j u^{ij}. \quad (4)$$

The maximax criterion (α = 0) is that of a dedicated optimist. This decision maker will make his decision solely on the basis of the highest return offered through each of decisions. He selects that action which will maximize his maximum possible return with no regard for possible consequences.

Savage minimax regret(risk) criterion. This criterion suggests that what we might really worry about is how bad we might feel afterwards when we see what we might have done if we had only known enough to do the right thing. What is regret? The «regret» is determined for each state of nature by subtracting the largest utility in each column of the decision model (in matrix form) from all other utilities in that column:

$$r_{ij} = \max_j u_{ij} — u_{ij}. \quad (5)$$

The decision maker then applies the minimax principle by selecting the alternative with the smallest maximum, i.e., the lowest value of the worst regret (risk):

$$S = \min_i \max_j r_{ij}. \quad (6)$$

As a final comment we would like to note that, in general, for the same decision situations, different decision criteria will result in different courses of action to be selected. Different decision criteria may be optimal for different decision situations and it is not always easy to select the «right» criterion for a given decision problem. Thus a decision criterion for decision criteria is needed and classical decision theory gives us relatively little guidance in this problem.

To sum up, uncertainty is an important factor in decisions but there is no unique method of dealing with uncertainty. There are several ways of making decisions under the condition of uncertainty. None of the methods as described above lead to a flawless decision. However, they do add some degree of certainty to decision-making. The choice of method depends on the availability of necessary data and reliability of a method under different conditions.

We have seen that a decision maker who acts according to classical decision theory would proceed as follow. First, a decision model is constructed. Next, a decision optimization criterion is established. Finally, a course of action is chosen according to the selected decision rule.

At the outset it must be emphasized that the study and application of classical decision theory does not add to the amount of information available to the decision maker. In other words, it is not the purpose of classical decision theory to remove or reduce uncertainty from the decision process.

Classical decision theory suffers from three major limitations: mathematical, methodological, and pragmatic.

In principle, decision theory has some approaches available for almost any type of decision problem. In practice, however, the classical approaches become computationally complex when the number of decision variables become large. In other approaches, the decision model (in form of playoff, loss or regret matrix) is usually an inaccurate representation of reality when a large number of state c is involved. Other difficulties of classical theory include:

- There seems to be a formal mechanism for handling those state variables which are fuzzy in nature and cannot be easy quantified.

- There are no adequate means for handling those problems where the objective function cannot be readily expressed in utility values or where the goals are fuzzy and exactly specified.

- No theory is available for filtering out relevant information for decision making from irrelevant data, except in very special cases.

The methodological difficulty encountered in applying classical decision theory techniques is that decision theory leaves out the formulation of an explicit decision model from the decision process. It tries to jump directly from the real-life situation to mathematical model, without investigating the nature of the mechanism of which the mathematical model is supposed to be a representation. A related problem is that of selecting a suitable decision criterion as we mentioned earlier. When facing structured uncertainty, for example, the decision maker has a choice of four criteria: Laplace (rational), Hurwicz (adventurous), Wald (cautious) and Savage (bad loser). Clearly, different decision criteria are optimal for different situations.

Perhaps the most serious deficiency of classical decision theory is that its decision schemes are static rather then dynamic. There are no formal ways of gradually improving or modifying the decision mechanism in the light of new information, as this information becomes available. Therefore, decision schemes developed with the use of orthodox decision theory are usually inefficient and suboptimal, and there is no way of optimizing them.

Observation of the managerial decision process suggests that its aim is to improve matters steadily, rather than to seek a rigorous optimum initially. The reason for this is clear. The decision maker is not at all sure that he can find in his own understanding a conceptual model which really represents the situation he is trying to control, nor that he can specify the relationship within it, nor that he knows all the criteria of success.

By the way, any decisions developed on the basis of the analysis or the forecast better then decision made spontaneously, at random.

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